Optimization Application: Algae Growth

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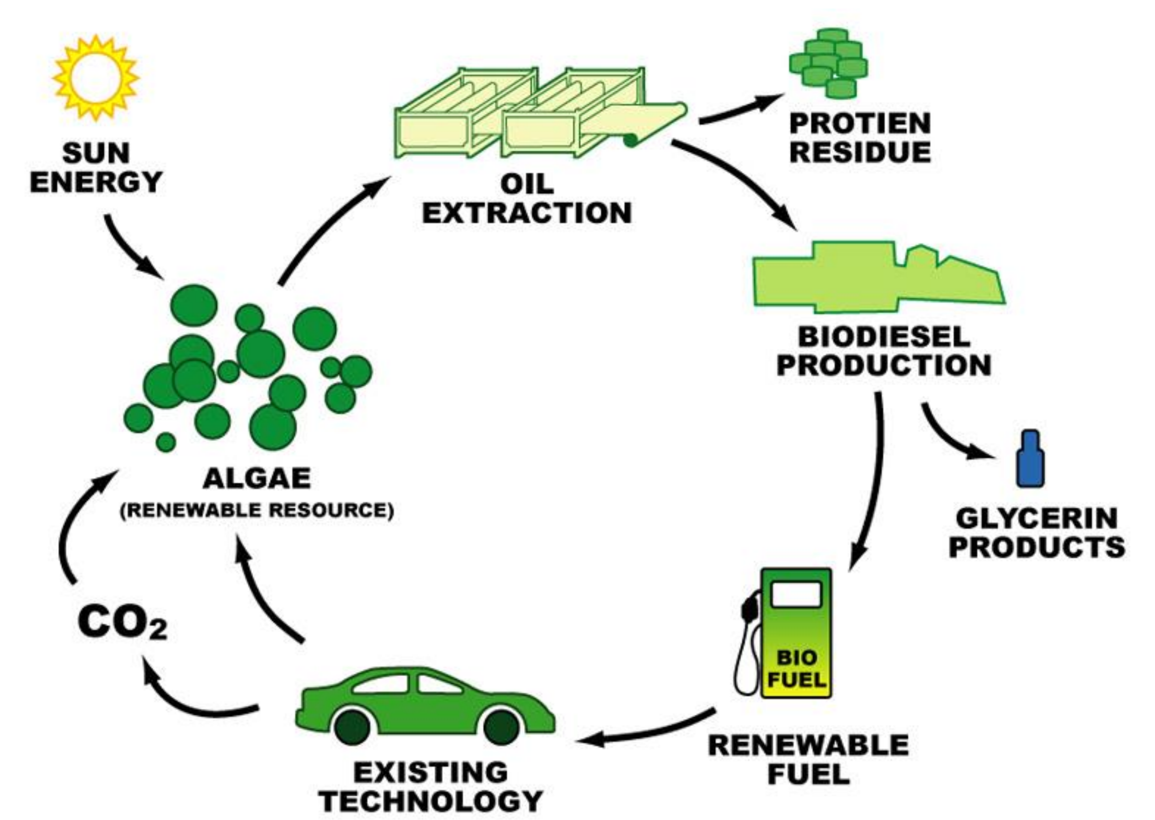
Objective

The goal of this project is to develop a mathematical model for the lipid production from nitrogen-limited algae grown in a homogenous photo bioreactor. This mathematical model will allow for regression and optimization procedures that will show the best parameters for lipid production to be applied to an industrial algae farm where the lipids may be harvested for biofuels.

Introduction

As the focus on alternative, sustainable energy sources continues to rise, optimization techniques are essential for the efficacy and economic viability of such energy sources to be harnessed. For optimization techniques to be utilized, a mathematical model of the system is necessary. One such renewable energy source is that of biofuels produced from the lipids of algae. To make this system a fiscally profitable endeavor, many parameters of algae growth must be considered and optimized. This project focused on two such parameters: the effect of nitrogen stressed lipid growth in algae and a more representative death rate of algae.



Approach

The first feature of lipid production that this project focuses on is the effect of nitrogen limitation on algae growth. This lipid production model was taken from a preexisting model and then coefficients were found heuristically by graphical analysis. The second feature is a death rate of algae, which was a development from another model that focused on the fact that not all the algae will live to the harvest time. The development of the model is derived from the following growth parameters:

(1) $$L \xrightarrow{k1*P1} L + L$$

(2) $$L \xrightarrow{k2*P2} D$$

Algae – Growth Model

The development of the model is as follows:

(3) $$\rho(s) = \rho m * \left(\frac{n}{n+Kn}\right)$$

(4) $$\mu(qn) = \bar{\mu} * \left(1 - \frac{Q_0}{qn}\right)$$

(5) $$n_{new} = n_{old} + \Delta T * (\delta * n_0 - \rho(s) * L - \delta * n)$$

(6) $$\ell_{new} = \ell_{old} + \Delta T * (\beta * qn * \mu(qn) * L - \gamma * \rho(s) * L - \delta * \ell)$$

(7) $$qn_{new} = qn_{old} + \Delta T * (\rho(s) - \mu(qn) * qn)$$

(8) $$L_{new} = L_{old} + \Delta T * (P_1 * k_1 * L - k_2 * L - \delta * L)$$

(9) $$D_{new} = D_{old} + \Delta T * (k_2 * P_2 * L - \delta * D)$$

(10) $$\ell_{fraction} = \frac{\ell}{L}$$

(11) $$\ell_{dead} = \ell_{fraction} * D$$

(12) $$\ell_{total} = \left(L * \ell_{fraction} + \sum \ell_{dead}\right) * V$$

(13) $$V = V_0 + v_0 * T$$

(14) $$\ell_{yearly} = \frac{365.25 * \ell_{total}}{T + T_{turnover}}$$

$L \equiv live\ algae\ concentration \left[\frac{mg\ C}{L}\right]$

$D \equiv dead\ algae\ concentration \left[\frac{mg\ C}{L}\right]$

$k1 \equiv live\ algae\ rate\ constant\ [d^{-1}]$

$k2 \equiv dead\ algae\ rate\ constant\ [d^{-1}]$

$P1 \equiv live\ algae\ production\ coefficient$

$P2 \equiv dead\ algae\ lipid\ coefficient$

$\rho(n) \equiv absorption\ of\ nitrogen \left[\frac{mg\ N}{mg\ C * d}\right]$

$\rho m \equiv maximum\ uptake\ rate \left[\frac{mg\ N}{mg\ C * d}\right]$

$n \equiv nitrogen\ concentration \left[\frac{mg\ N}{L}\right]$

$Kn \equiv half-saturation\ constant \left[\frac{mg\ N}{L}\right]$

$\mu(qn) \equiv specific\ rate\ of\ growth\ [d^{-1}]$

$\bar{\mu} \equiv theoretical\ maximum\ growth\ [d^{-1}]$

$Q_0 \equiv minimum\ nitrogen\ quota \left[\frac{mg\ N}{mg\ C}\right]$

$qn \equiv nitrogen\ quota \left[\frac{mg\ N}{mg\ C}\right]$

$T \equiv time\ [d]$

$\Delta T \equiv change\ in\ time\ [d]$

$\delta \equiv dilution\ rate\ [d^{-1}]$

$n_0 \equiv influent\ nitrogen\ concentration \left[\frac{mg\ N}{L}\right]$

$l \equiv lipid\ concentration \left[\frac{mg\ C}{L}\right]$

$\beta \equiv fatty\ acid\ synthesis\ coefficient \left[\frac{mg\ C}{mg\ N}\right]$

$\gamma \equiv fatty\ acid\ mobilization\ coefficient \left[\frac{mg\ C}{mg\ N}\right]$

$\ell_{fraction} \equiv fraction\ of\ live\ algae\ that\ is\ lipid \left[\frac{mg\ C}{mg\ C}\right]$

$\ell_{dead} \equiv lipid\ concentration\ from\ dead\ algae \left[\frac{mg\ C}{L}\right]$

$\ell_{total} \equiv total\ lipid\ production\ [mg\ C]$

$depth \equiv depth\ of\ reactor\ [m]$

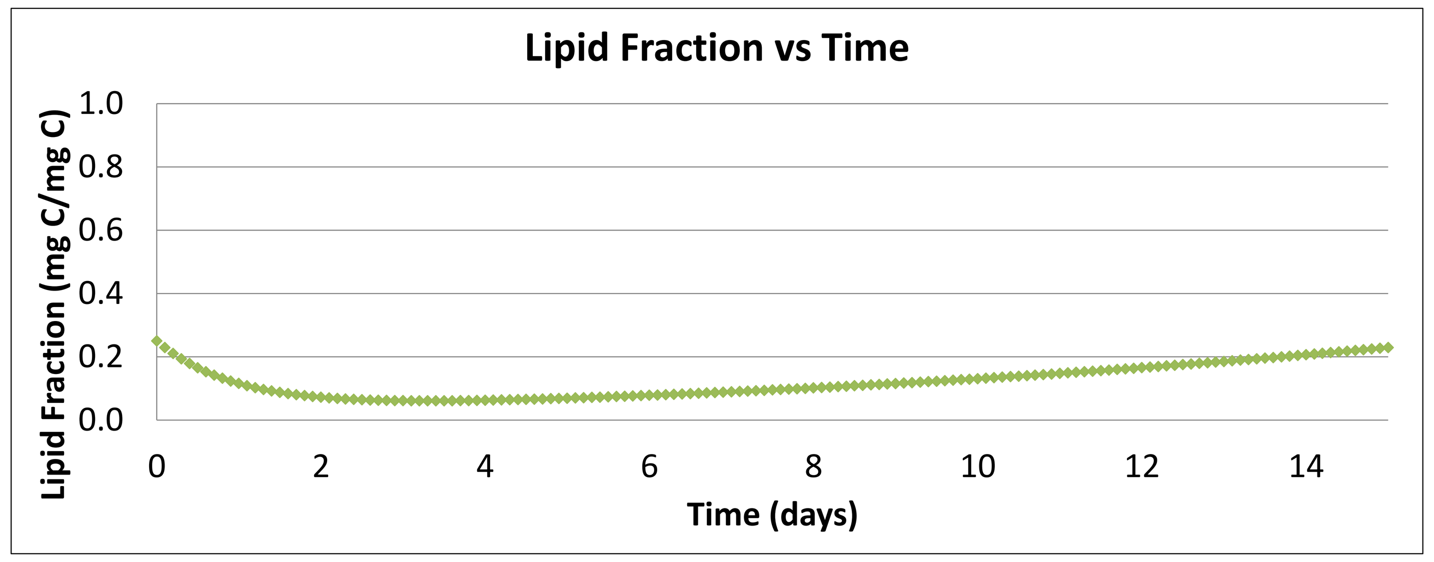
$\ell_{yearly} \equiv yearly\ lipid\ production\ [mg\ C]$

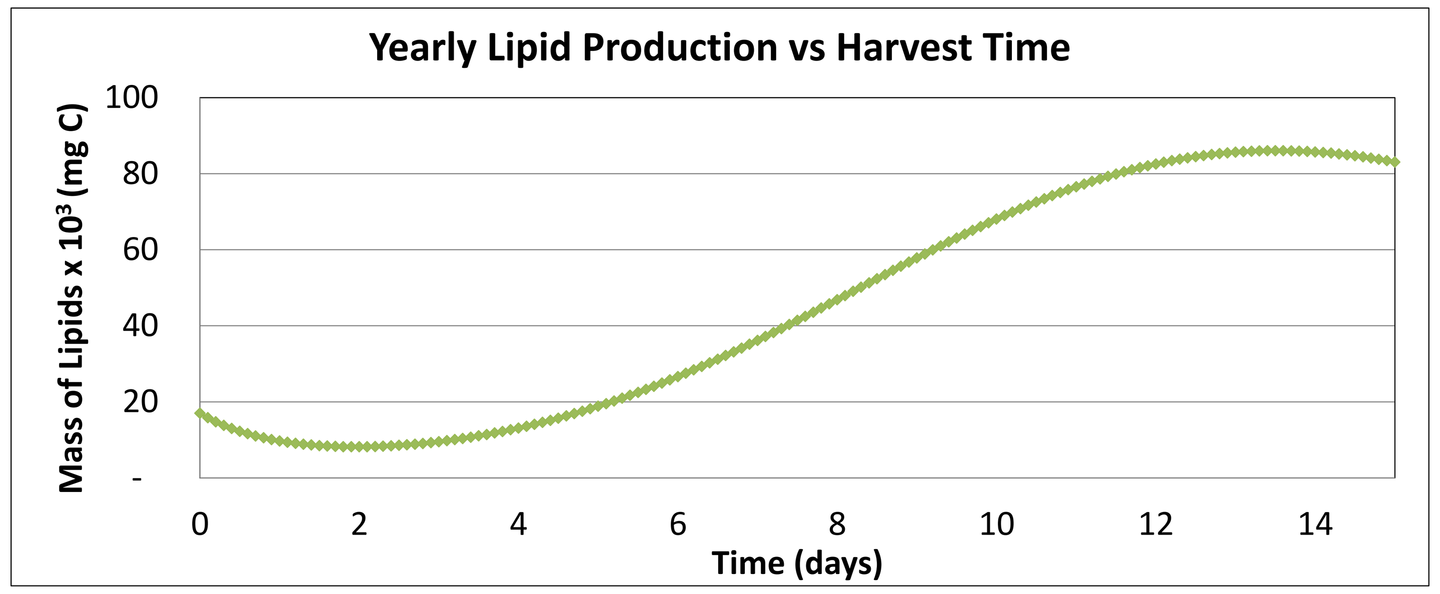
$T_{turnover} \equiv time\ to\ harvest\ and\ set\ up\ system\ [d]$

$V \equiv volume\ of\ reaction\ [L]$

$v_0 \equiv volumetric\ flowrate\ into\ reactor\ [\frac{L}{d}]$

Graphical Analysis





Conclusion

As seen from the graphs above, a model was developed for the production of lipids from algae that analyzed the effect of limiting the nitrogen of the system and incorporated a more representative expression of algae growth in terms of death rate. After the development of the model, a heuristic approach for the graphical analysis led to the coefficients on the right, with an optimal harvest time of 13.5 days to maximize yearly lipid production as seen in Equation (14). It is recommended that a better mathematical model for lipid production be determined from a biology standpoint. It is important to note that equations are sensitive to n_0. This model should serve as a building block to further research on lipids produced from algae to be harnessed for biofuels.

Q_0	0.0001	β	400
k1	0.36	ϒ	0.1
k2	0.00005	Kn	10
P2	0.1	$\bar{\mu}$	0.001
Depth	0.8	ρm	0.05
qn_0	0.100	δ	0.4
$T_{turnover}$	2	P1	0.8
Reactor Diameter (m)			7.700
Reactor Area (m^2)			46.566
V_0 (L)			37253.0
V_0 (L/d)			10
Depth Increase (m/d)			0.00021

References

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